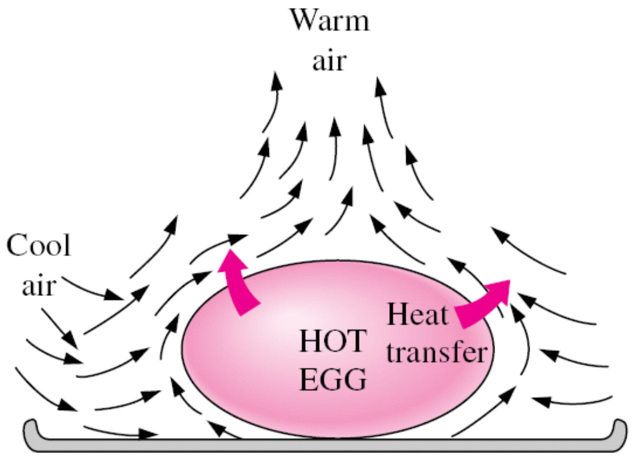
Objectives

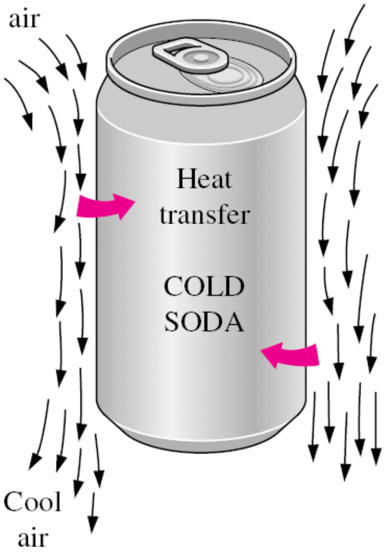
- Understand the physical mechanism of natural convection
- Evaluate the Grashof number
- Evaluate the Nusselt number for natural convection associated with vertical, horizontal, and inclined plates as well as cylinders and spheres
- Examine natural convection from finned surfaces, and determine the optimum fin spacing
- Analyze natural convection inside enclosures such as double-pane windows
- Consider combined natural and forced convection, and assess the relative importance of each mode.

Natural Convection

- **Examples:**
  - Heat transfer from electric baseboard heaters
  - Heat transfer from refrigeration coils
  - Heat transfer from our body

- Natural convection in gases usually accompanied by radiation of comparable magnitude





Natural Convection

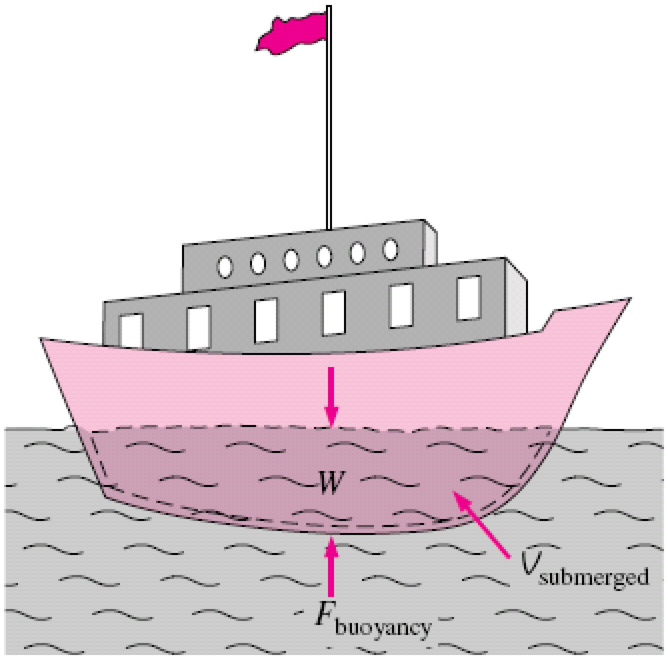
- Buoyancy forces are responsible for the fluid motion in natural convection.
- Viscous forces oppose the fluid motion.

- In gravitational field, the upward force exerted by a fluid on a body completely or partially immersed in it → **buoyancy force**

$$F_{\text{buoyancy}} = \rho_{\text{fluid}}\, g V_{\text{body}}$$

Net force:

$$F_{\text{net}} = W - F_{\text{buoyancy}}$$
$$= \rho_{\text{body}}\, g V_{\text{body}} - \rho_{\text{fluid}}\, g V_{\text{body}}$$
$$= (\rho_{\text{body}} - \rho_{\text{fluid}})\, g V_{\text{body}}$$

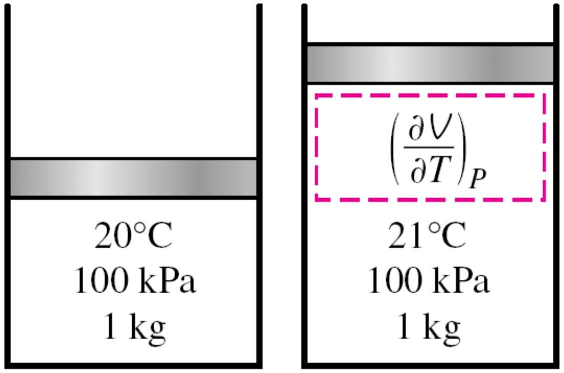


Natural Convection

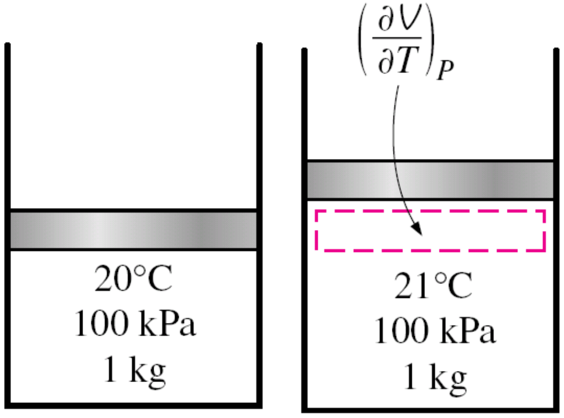
- In heat transfer → express the net buoyancy force in terms of temperature difference

- Buoyancy forces are expressed in terms of fluid temperature differences through the:
  volume expansion coefficient

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)\bigg|_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)\bigg|_P \quad (1/K)$$



(*a*) A substance with a large β



(*b*) A substance with a small β

Volume expansion coefficient β

- The volume expansion coefficient can be expressed approximately by replacing differential quantities by differences as

$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T} \qquad (\text{at constant } P) \qquad \textbf{(11-4)}$$

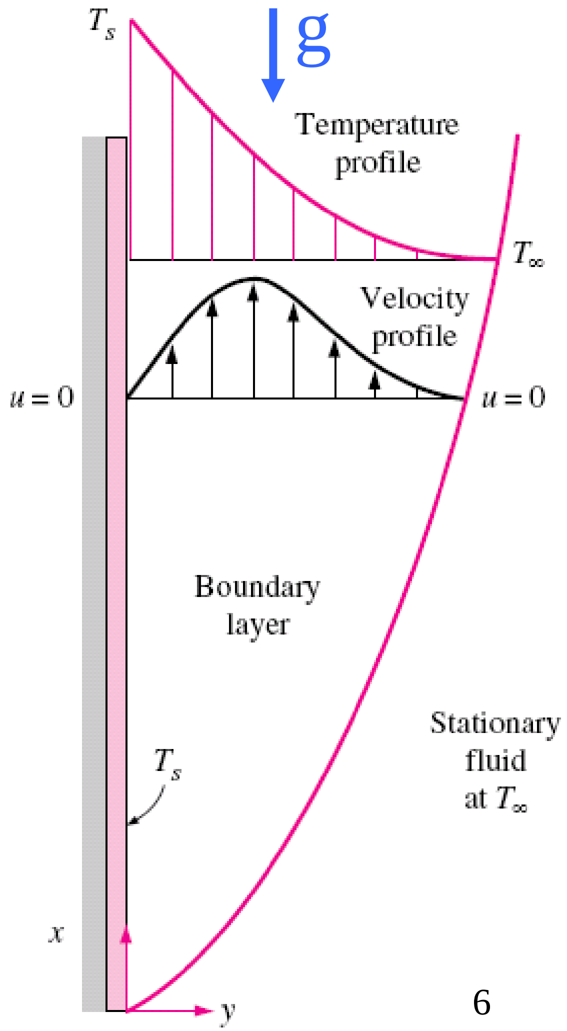
$$\rho_\infty - \rho = \rho\beta\left(T - T_\infty\right) \qquad (\text{at constant } P) \qquad \textbf{(11-5)}$$

- For *ideal gas*

$$\boxed{\beta_{\text{ideal gas}} = \frac{1}{T}} \qquad (1/\text{K}) \qquad \textbf{(11-6)}$$

Equation of Motion and the Grashof Number

- Consider a vertical hot flat plate immersed in a quiescent fluid body.
- Assumptions:
  - steady,
  - laminar,
  - two-dimensional,
  - Newtonian fluid, and
  - constant properties, except the density difference $\rho - \rho_\infty$ (Boussinesq approximation).



Consider a differential volume element.

- Newton's second law of motion

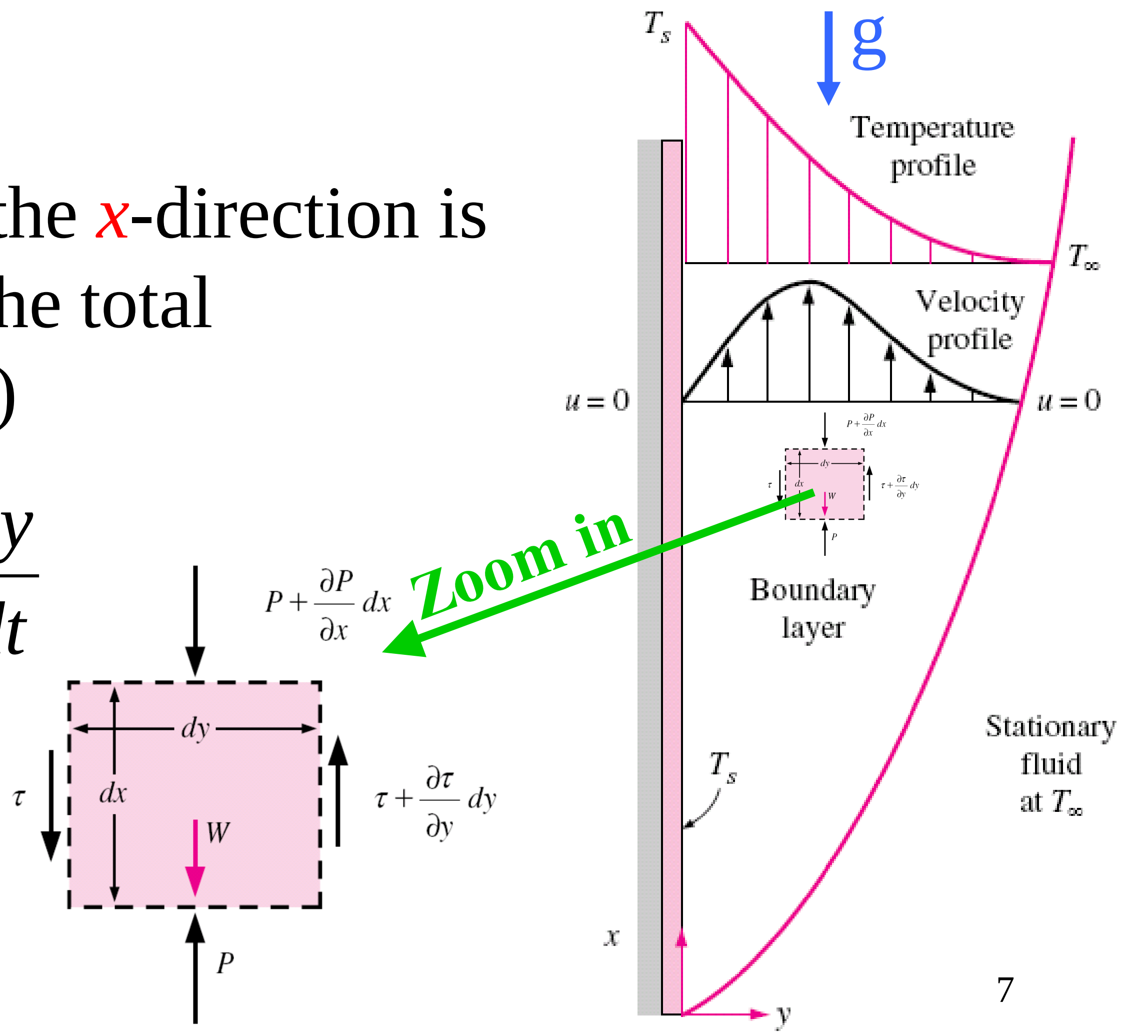
$$\delta m \cdot a_x = F_x \qquad \textbf{(11-7)}$$

$$\delta m = \rho\left(dx \cdot dy \cdot 1\right)$$

- The acceleration in the x-direction is obtained by taking the total differential of $u(x, y)$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

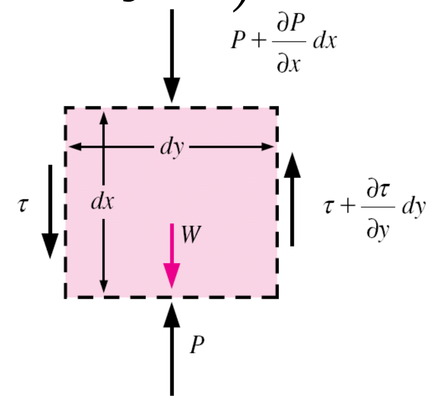
$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$



- The net surface force acting in the *x*-direction

$$F_x = \overbrace{\left(\frac{\partial \tau}{\partial y} dy\right)(dx \cdot 1)}^{\text{Net viscous force}} - \overbrace{\left(\frac{\partial P}{\partial x} dx\right)(dy \cdot 1)}^{\text{Net pressure force}} - \overbrace{\rho g\,(dx \cdot dy \cdot 1)}^{\text{Gravitational force}}$$

$$= \left(\mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g\right)(dx \cdot dy \cdot 1) \qquad \textbf{(11-9)}$$



- Substituting Eqs. 11–8 and 11–9 into Eq. 11–7 and dividing by $\rho \cdot dx \cdot dy \cdot 1$ gives the *conservation of momentum* in the *x*-direction

$$\boxed{\rho\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g} \qquad \textbf{(11-10)}$$

- The x-momentum equation in the quiescent fluid outside the boundary layer (setting u=0)

$$\frac{\partial P_\infty}{\partial x} = -\rho_\infty g \qquad \textbf{(11-11)}$$

- Noting that
  - $v << u$ in the boundary layer and thus $\partial v/\partial x \approx \partial v/\partial y \approx 0$, and
  - there are no body forces (including gravity) in the y-direction,

the force balance in the y-direction is

$$\frac{\partial P}{\partial y} = 0 \Rightarrow P(x) = P_\infty(x) = P \Rightarrow \frac{\partial P}{\partial x} = \frac{\partial P_\infty}{\partial x} = -\rho_\infty g$$

Substituting into Eq. 9–10

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + \left(\rho_\infty - \rho\right)g \qquad \textbf{(11-12)}$$

- Substituting Eq. 11-5 it into Eq. 11-12 and dividing both sides by ρ gives

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta\left(T-T_\infty\right) \qquad \textbf{(11-13)}$$

- The momentum equation involves the temperature, and thus the momentum and energy equations must be solved simultaneously.

- The set of three partial differential equations (the continuity, momentum, and the energy equations) that govern natural convection flow over vertical isothermal plates can be reduced to a set of two ordinary nonlinear differential equations by the introduction of a similarity variable.

The Grashof Number

- The governing equations of natural convection and the boundary conditions can be nondimensionalized

$$x^* = \frac{x}{L_c} \;;\; y^* = \frac{y}{L_c} \;;\; u^* = \frac{u}{V} \;;\; v^* = \frac{v}{V} \;;\; T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

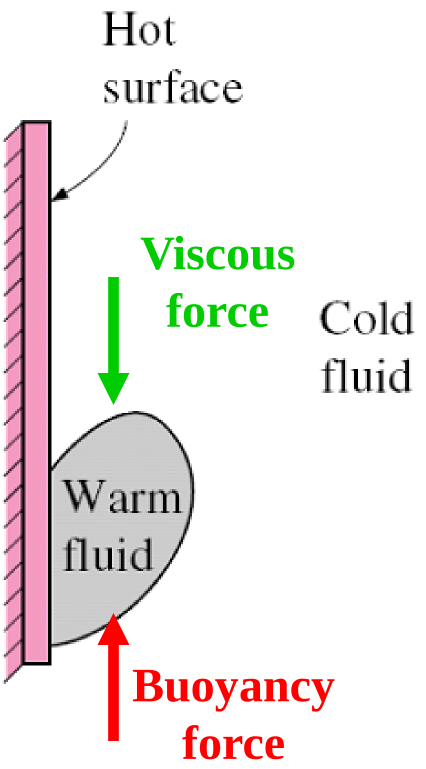
- Substituting into the momentum equation and simplifying give

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \underbrace{\left[\frac{g\beta\left(T_s - T_\infty\right)L_c^3}{\nu^2} \right]}_{Gr_L} \frac{T^*}{\mathrm{Re}_L^2} + \frac{1}{\mathrm{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}} \quad \textbf{(11-14)}$$

- The dimensionless parameter in the brackets represents the natural convection effects, and is called the **Grashof number** Gr_L

$$Gr_L = \frac{g\beta\left(T_s - T_\infty\right)L_c^3}{\nu^2} \qquad \textbf{(11-15)}$$

$$Gr_L = \frac{\textit{Buoyancy force}}{\textit{Viscous force}}$$



- The flow regime in natural convection is governed by the *Grashof number*

$Gr_L > 10^9$ flow is turbulent

Natural Convection over Surfaces

- Natural convection heat transfer on a surface depends on
  - geometry,
  - orientation,
  - variation of temperature on the surface, and
  - thermophysical properties of the fluid.
- The simple empirical correlations for the average *Nusselt number* in natural convection are of the form

$$Nu = \frac{hL_c}{k} = C \cdot \left(Gr_L \cdot \Pr\right)^n = C \cdot Ra_L^n \quad \textbf{(11-16)}$$

- Where Ra_L is the **Rayleigh number**

$$Ra_L = Gr_L \cdot \Pr = \frac{g\beta\left(T_s - T_\infty\right)L_c^3}{\nu^2}\Pr \quad \textbf{(11-17)}$$

- The values of the constants *C* and *n* depend on the *geometry* of the surface and the *flow regime* (which depend on the Ra).
- All fluid properties are to be evaluated at the film temperature $T_f=(T_s+T_\infty)$.

- Nu relations for constant T_s are applicable for the case of constant q_s, but the plate midpoint temperature $T_{L/2}$ is used for T_s in the evaluation of the film temperature.

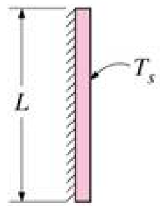
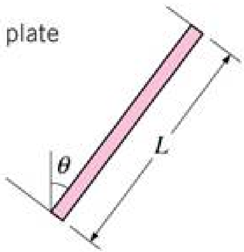
- Thus for uniform heat flux:

$$Nu = \frac{hL}{k} = \frac{\dot{q}_s L}{k\left(T_{L/2} - T_\infty\right)} \qquad \textbf{(11-27)}$$

Empirical correlations for Nu_{avg}

TABLE 9–1

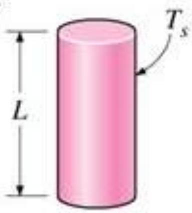
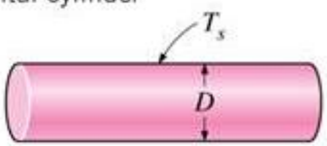
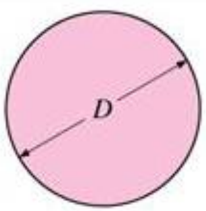
Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu	
Vertical plate 	L	10^4–10^9 10^{20}–10^{13} Entire range	$\text{Nu} = 0.59\text{Ra}_L^{1/4}$ $\text{Nu} = 0.1\text{Ra}_L^{1/3}$ $\text{Nu} = \left\{0.825 + \dfrac{0.387\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (complex but more accurate)	**(9–19)** **(9–20)** **(9–21)**
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos\theta$ for $\text{Ra} < 10^9$	
Horizontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate)  (b) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\text{Nu} = 0.54\text{Ra}_L^{1/4}$ $\text{Nu} = 0.15\text{Ra}_L^{1/3}$ $\text{Nu} = 0.27\text{Ra}_L^{1/4}$	**(9–22)** **(9–23)** **(9–24)**

Empirical correlations for Nu_{avg}

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{\text{Gr}_L^{1/4}}$	
Horizontal cylinder 	D	$\text{Ra}_D \leq 10^{12}$	$\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$	**(9–25)**
Sphere 	D	$\text{Ra}_D \leq 10^{11}$ $(\text{Pr} \geq 0.7)$	$\text{Nu} = 2 + \frac{0.589\text{Ra}_D^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$	**(9–26)**

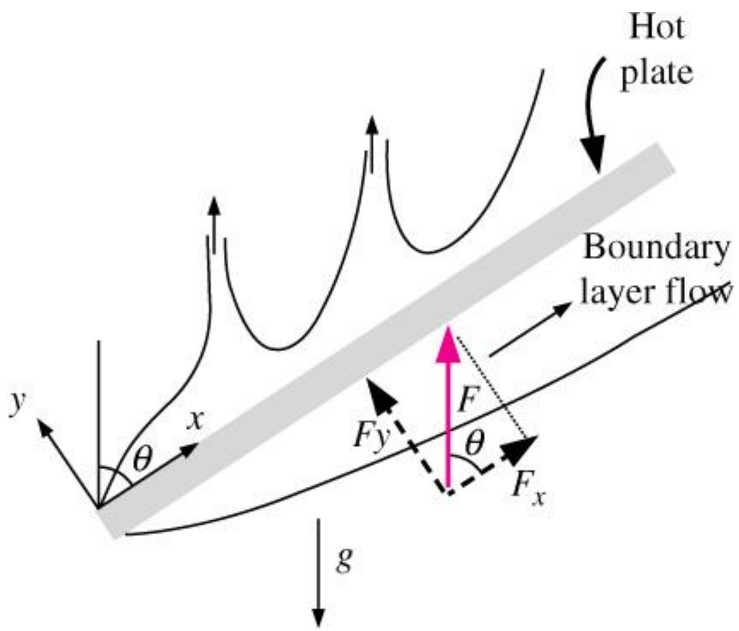


FIGURE 9–10
Natural convection flows on the upper and lower surfaces of an inclined hot plate.

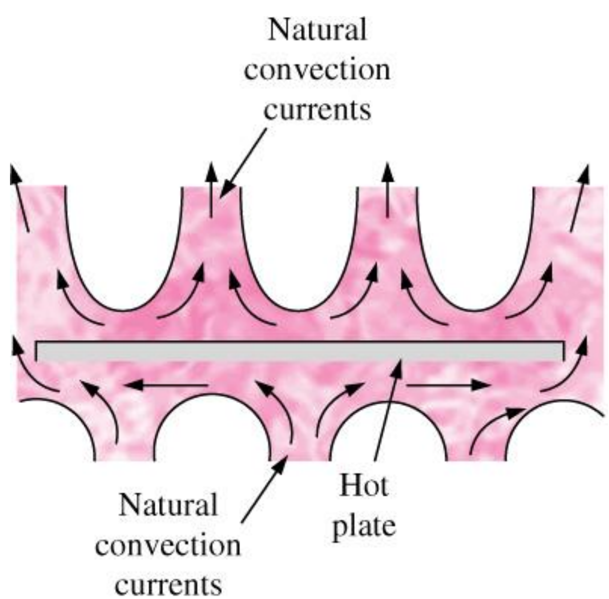


FIGURE 9–11
Natural convection flows on the upper and lower surfaces of a horizontal hot plate.

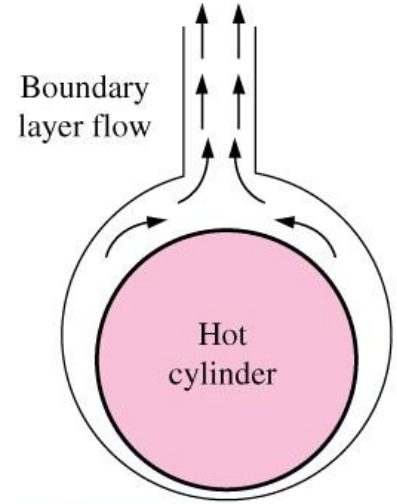


FIGURE 9–12
Natural convection flow over a horizontal hot cylinder.

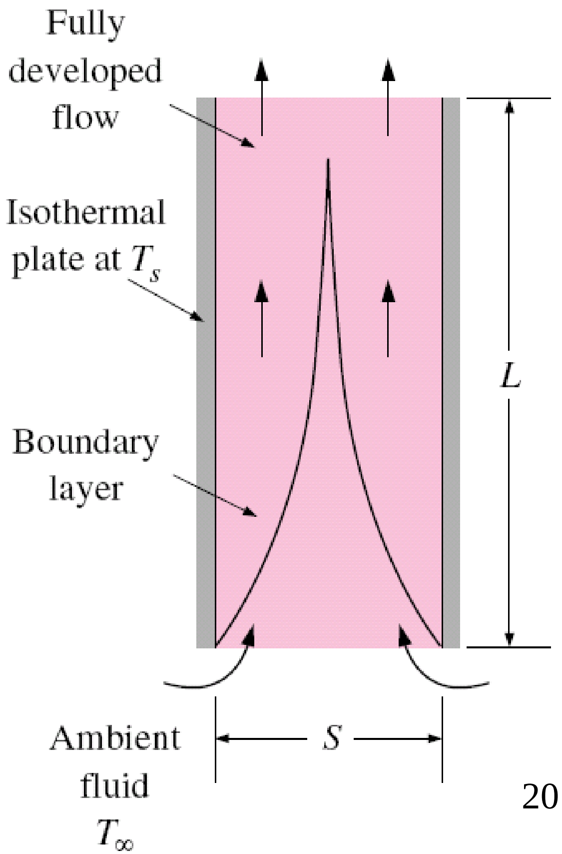
Review of Last Monday

- Driving force of natural convection?
- Volume expansion coefficient?
- Temperature and velocity profiles?
- Grashof number? Rayleigh number?
- Nusselt number relations?

9-97 A vertical cylindrical pressure vessel is 1.0 m in diameter and 3.0 m in height. Its outside average wall temperature is 60°C, while the surrounding air is at 0°C. Calculate the rate of heat loss from the vessel's cylindrical surface when there is (*a*) no wind and (*b*) a crosswind of 20 km/h.

Natural Convection from Finned Surfaces

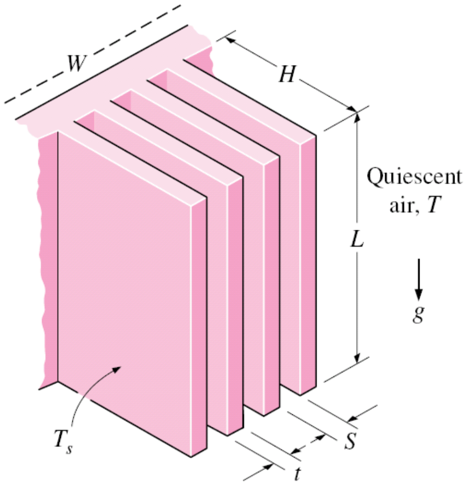
- Natural convection flow through a channel formed by two parallel plates is commonly encountered in practice.
- Long Surface
  - fully developed channel flow.
- Short surface or large spacing
  - natural convection from two independent plates in a quiescent medium.



- The recommended relation for the average Nusselt number for vertical isothermal parallel plates is

$$Nu = \frac{hS}{k} = \left[\frac{576}{\left(Ra_s\, S/L\right)^2} + \frac{2.873}{\left(Ra_s\, S/L\right)^{0.5}} \right]^{-0.5} \quad \textbf{(11-31)}$$

- Closely packed fins
  - greater surface area
  - smaller heat transfer coefficient.
- Widely spaced fins
  - higher heat transfer coefficient
  - smaller surface area.

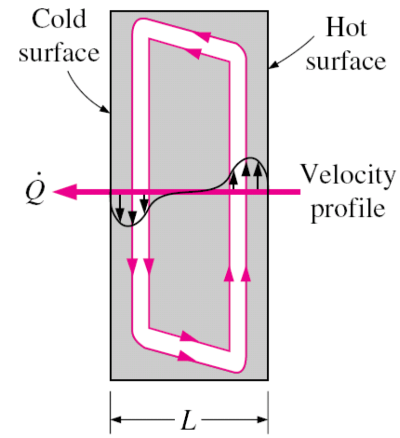


- Optimum fin spacing for a vertical heat sink

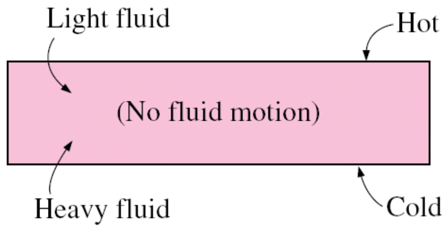
$$S_{opt} = 2.714\left(\frac{S^3 L}{Ra_s}\right)^{0.25} = 2.714\frac{L}{Ra_L^{0.25}} \quad \textbf{(11-32)}$$

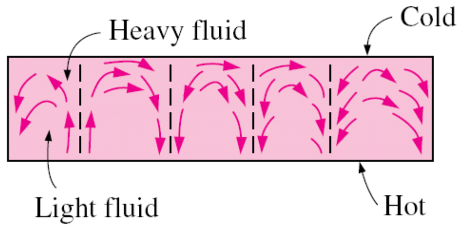
Natural Convection Inside Enclosures

- In a vertical enclosure, the fluid adjacent to the hotter surface rises and the fluid adjacent to the cooler one falls, setting off a rotationary motion within the enclosure that enhances heat transfer through the enclosure.



- Heat transfer through a horizontal enclosure
  - hotter plate is at the top ─ no convection currents (Nu=1).
  - hotter plate is at the bottom
    - Ra<1708 no convection currents (Nu=1).
    - $3\text{x}10^5$>Ra>1708 Bénard Cells.
    - Ra>$3\text{x}10^5$ turbulent flow.





Nusselt Number Correlations for Enclosures

- Simple power-law type relations in the form of

$$Nu = C \cdot Ra_L^n$$

where *C* and *n* are constants, are sufficiently accurate, but they are usually applicable to a narrow range of Prandtl and Rayleigh numbers and aspect ratios.

- Numerous correlations are widely available for
  - horizontal rectangular enclosures,
  - inclined rectangular enclosures,
  - vertical rectangular enclosures,
  - concentric cylinders,
  - concentric spheres.

Combined Natural and Forced Convection

- Heat transfer coefficients in forced convection are typically much higher than in natural convection.
- The error involved in ignoring natural convection may be considerable at low velocities.

- Nusselt Number:
  - Forced convection (flat plate, laminar flow):

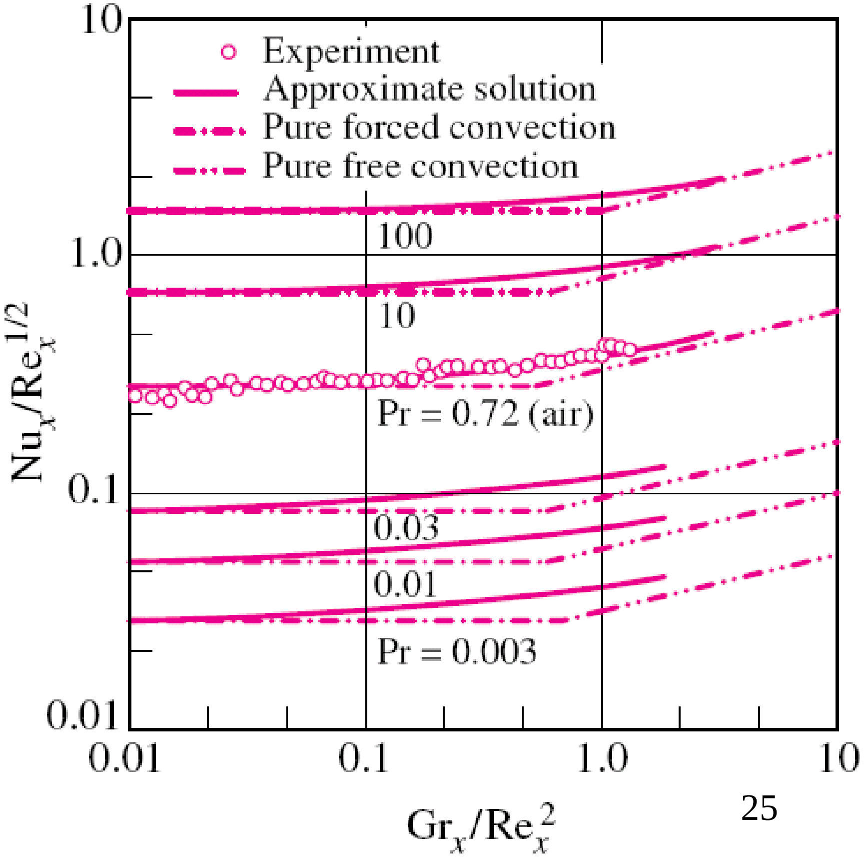
$$Nu_{\text{forced convection}} \propto \mathrm{Re}^{1/2}$$

  - Natural convection (vertical plate, laminar flow):

$$Nu_{\text{natural convection}} \propto Gr^{1/4}$$

- The parameter Gr/Re^2 represents the importance of natural convection relative to forced convection.

- $Gr/Re^2 < 0.1$
  - natural convection is negligible.
- $Gr/Re^2 > 10$
  - forced convection is negligible.
- $0.1 < Gr/Re^2 < 10$
  - forced and natural convection are not negligible.



- Natural convection may *help* or *hurt* forced convection heat transfer

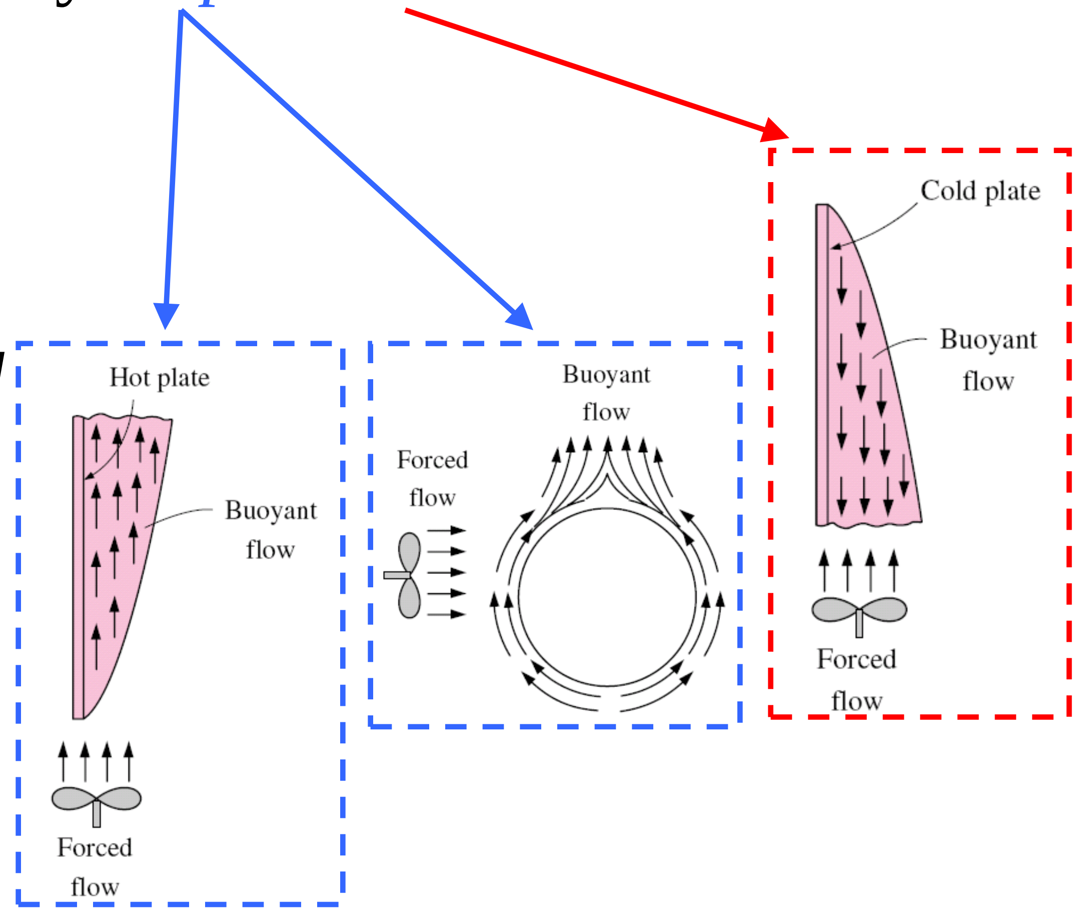
  depending on the

  relative directions

  of *buoyancy-induced*

  and the *forced*

  convection motions.



Nusselt Number for Combined Natural and Forced Convection

- A review of experimental data suggests a Nusselt number correlation of the form

$$Nu_{\text{combined}} = \left(Nu^n_{\text{forced}} \pm Nu^n_{\text{natural}} \right)^{1/n} \qquad \textbf{(11-66)}$$

$$n \sim 3 - 4$$

- $\text{Nu}_{\textit{forced}}$ and $\text{Nu}_{\textit{natural}}$ are determined from the correlations for *pure forced* and *pure natural convection*, respectively.

9-79 In a production facility, thin square plates 2 m × 2 m in size coming out of the oven at 270°C are cooled by blowing ambient air at 18°C horizontally parallel to their surfaces. Determine the air velocity above which the natural convection effects on heat transfer are less than 10 percent and thus are negligible.